Differences in Measurement Uncertainty Between Horizontal and Elevation Angles

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Key words: total station, angle measurement, uncertainty

SUMMARY

As introduced in Murakami (FIG WW2023), we are conducting research toward modernizing the General Standard of Operation Specifications for Public Surveys. Within this research, we investigated the angular measurement uncertainty of total stations (TS). While we initially assumed the measurement uncertainties for horizontal and elevation angles were equal, the results suggest they are not.
From surveying instrument inspection data, we investigated the angular measurement uncertainty of TS units with 1-second and 5-second readings.
The measurement uncertainty for horizontal angles was:
 1.1 seconds per sighting, in terms of standard deviation for 1-second reading;
 1.7 seconds per sighting, in terms of standard deviation for 5-second reading.
The measurement uncertainty for elevation angles was:
 1.8 seconds per sighting, in terms of standard deviation for 1-second reading;
 2.4 seconds per sighting, in terms of standard deviation for 5-second reading.
The measurement uncertainty for elevation angles is approximately 1.5 times that for horizontal angles.
Since we estimate TS instruments are most frequently set to 5-second reading in field works, TS was set to 5-second reading for the field experimental observations. From the field experiment data, we also investigated the uncertainties.
The measurement uncertainty for horizontal angles was:
 2.5 seconds per sighting, in terms of standard deviation.
The measurement uncertainty for elevation angles was:
 5.0 seconds per sighting, in terms of standard deviation.
The measurement uncertainty for elevation angles is twice that for horizontal angles.
The reasons why the measurement uncertainty of elevation angles is greater than that of horizontal angles, and why the measurement uncertainty in experimental observations is greater for both elevation and horizontal angles than that in instrument inspections, are currently under investigation.

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1. INTRODUCTION

The Japan Association of Surveyors, a public interest incorporated association, established a study group on modernizing the operation specification for public surveys in August 2021 (Murakami, 2023). This initiative, undertaken from a private-sector perspective to modernize the General Standard of Operation Specifications for Public Surveys (GSI, 2025), has advanced discussions for formulating new specifications with the cooperation of researchers and engineers from universities and companies. The modernization research aims to develop specifications that reflect current surveying technology and instrument capabilities, establishing revised quality control standards and methods. As a first step in modernization, the angular measurement uncertainty of total stations (TS) was estimated based on data from surveying instrument inspections and field experiments.

2. Evaluation of Total Station Performance Based on Surveying Instrument Inspection Data

2.1 Overview of Surveying Instrument Inspection

The angular measurement uncertainty of total stations (TS) was estimated based on instrument inspection data. The Geospatial Information Authority of Japan and most public entities require contracted surveying companies to undergo third-party inspection of surveying instruments at least once a year as a principle. The Japan Association of Surveyors is one of the organizations capable of conducting this inspection and performs instrument inspections for surveying companies upon request. This inspection data revealed the actual usage status and performance of instruments in current public surveying.
The inspection data used was from 1,052 TS units inspected in fiscal year 2019. Among these, the most common setting for angular measurement was a minimum reading unit of 5 seconds, applied to 585 units. The next most common setting was 1 second, applied to 343 units; of these, one unit lacked elevation angle inspection data.
Murakami (2024) reported on the uncertainty of horizontal angles for TS units with a 5-second reading unit. This section adds to that by reporting on the uncertainty of horizontal

angles with a 1-second reading unit and the uncertainty of elevation angles with both 5-second and 1-second reading units.

2.1.1 Observation of Horizontal Angles in Inspection

As reported by Murakami (2024), the magnitude of the double angle difference, the observation difference, and the set-to-set difference is calculated using observations in three directions (0°, 90°, 180° horizontally) with two sets of three paired observations (Figure 1).

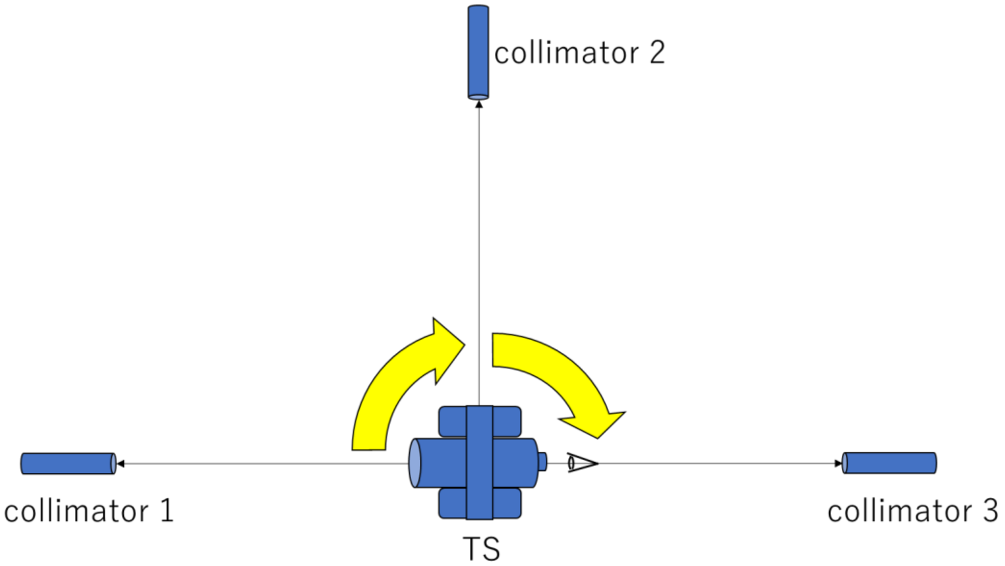

Figure 1. Observation of horizontal angles during inspection

The definitions of the double angle difference and the observation difference are as follows: The difference between the respective angles formed in the telescope's normal and inverted positions is called the angular discrepancy. The range (the difference between the maximum and minimum values) of angular discrepancies obtained over multiple paired observations is called the observation difference. Furthermore, the sum of the respective angles is called the double angle, and the range of double angles obtained over multiple paired observations is called the double angle difference. Quality control of horizontal angle observations is performed based on the magnitude of the observation difference and the double angle difference.

2.1.2 Observation of Elevation Angles in Inspection

By conducting a single paired observation in three directions (elevation angles: -30°, 0°, +30°) (Figure 2), an altitude constant for each direction is calculated.
The altitude constant is defined as follows:

The angle obtained by subtracting 360 degrees from the sum of the zenith angles in both the telescope's normal and inverted positions. The range of altitude constants is called the altitude constant discrepancy, and its magnitude is used for quality control of elevation angle observations.

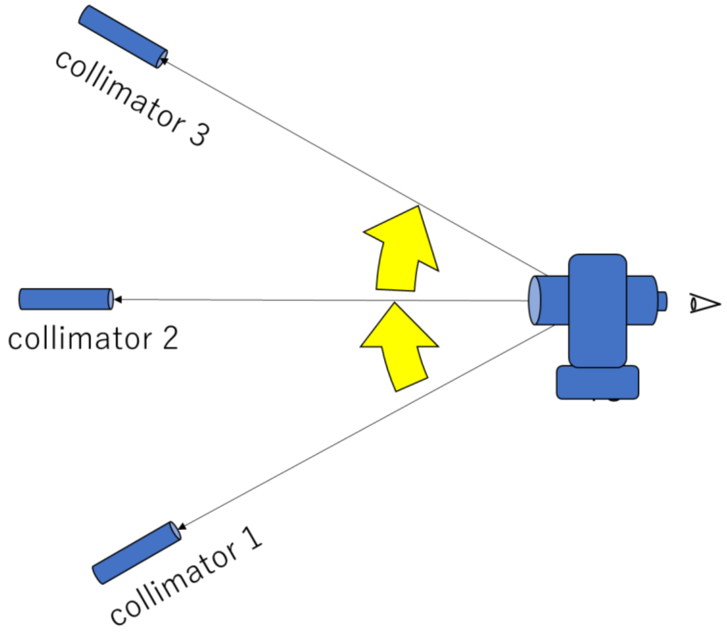

Figure 2. Observation of elevation angles during inspection

2.2 Angle Measurement Performance of Total Stations

2.2.1 Uncertainty in Horizontal Angle Sighting

We extracted and analyzed the set-to-set differences from the inspection data.
For 5-second reading TS, as reported by Murakami (2024), the average set-to-set difference in the inspection was 1.1 seconds, from which the estimated angular measurement uncertainty per sighting is 1.7 seconds. Furthermore, this analysis similarly yielded values of 0.74 seconds for the average set-to-set difference and 1.1 seconds per sighting for the 1-second reading TS. Additionally, for the 5-second reading TS, 99.96% of the observation differences fell within a range of 15 seconds, and 99.8% of the double angle differences fell within a range of 15 seconds. For the 1-second reading TS, 99.8% of both observation differences and double angle differences fell within a range of 12 seconds. Since both the angle discrepancy and double angle are obtained from four sightings each, their uncertainty σ_{diff} is twice the uncertainty σ_s of a single sighting. Since the observation difference and double angle difference represent the respective ranges of the angle discrepancy and double angle, the upper control limit U_{CL} for the range is determined by applying Shewhart control charts (JIS Z 9020-2, 2016) [Shewhart control charts (ISO 7870-2, 2013)],

$$U_{CL} = D_2 * \sigma_{diff} = D_2 * 2\ \sigma_s\ .$$

While the upper control limit U_{CL} is typically defined using the uncertainty σ, here we use a method opposite of the standard Shewhart control chart approach. Based on the double angle difference and observation difference obtained from the inspection data, the upper control limits are set as:

U_{CL} = 15" (5-second reading); U_{CL} = 12" (1-second reading)

to determine the uncertainty σ_s for a single sighting. Since D_2 = 4.358 (when n=3, k=3; n is the number of data, k is the control coefficient),

σ_s = 1.7" (5-second reading); σ_s = 1.4" (1-second reading).

For the 5-second reading TS, this is equal to σ_s derived from the set-to-set difference. For the 1-second reading TS, it is larger than σ_s derived from the set-to-set difference, but the order of magnitude is equivalent.

The above suggests that the uncertainty of a parameter can be estimated to some extent from the upper control limit of a certain parameter range.

2.2.2 Uncertainty in Elevation Angle Sighting

Since only the altitude constant discrepancy is recorded as inspection data for elevation angles, the maximum value of the altitude constant discrepancy was adopted as the upper control limit U_{CL}. Using the Shewhart control chart method, the uncertainty σ_K of the altitude constant and the resulting uncertainty σ_s of a single sighting are estimated. The relationship between the three parameters is:

$U_{CL} = D_2 * \sigma_K = D_2 * \sqrt{2} * \sigma_s$

Elevation angle inspection involves 585 5-second-reading TS units and 342 1-second-reading TS units. Observing the distribution of the respective altitude constant discrepancies, 100% of units fell within 15 seconds (5-second reading) and within 11 seconds (1-second reading). Considering these as upper control limits,

U_{CL} = 15" (5-second reading); U_{CL} = 11" (1-second reading)

and since D_2 = 4.358 (when n=3, k=3), we obtain:

σ_s = 2.4" (5-second reading); σ_s = 1.8" (1-second reading).

Although the reliability of the numerical values is not high due to the unconventional application of the Shewhart control chart method, it still suggests that the uncertainty in the elevation angle for a single sighting is 1.5 times larger than that in the horizontal angle.

3. Evaluation of Total Station Performance Based on Field Experiments

3.1 Experiment Overview

Since 5-second reading TS units are estimated to be the most commonly used in public surveying fieldwork, the field experiments employed three TS models set to 5-second reading. Horizontal angle observations and elevation angle observations were conducted at separate locations.

3.1.1 Horizontal Angle Observations

In the experimental observations conducted on February 3, 2021, at the calibration site of the baseline within the Geospatial Information Authority of Japan premises, observations were performed by two observers using three types of total stations to examine the effects of instrumental and individual errors (Murakami, 2024) (Figure 3). For horizontal angle observations, ten (10) observations were conducted, each consisting of two sets of three paired observations (telescope's normal and inverted positions). This yielded 120 samples for each of the two included angles (90° and 180°). During observations, the TS was fixed to a tribrach on a concrete pillar.

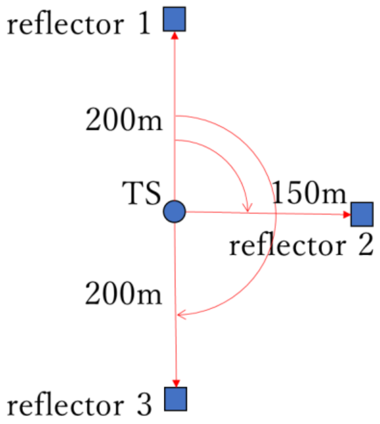

Figure 3. Horizontal Angle Observations in Field Experiments

3.1.2 Elevation Angle Observations

In the experimental observations conducted on June 25, 2024, at the parking lot of the Surveying Technology Center, Japan Association of Surveyors, three observers used three total stations to perform observations in order to examine the effects of instrumental and individual errors. A total of 80 readings were taken, comprising 10 sets of paired observations each targeting four sighting points installed on a building (Figure 4). This observation was repeated three times, alternating observers and total stations, resulting in a total of 240 readings. The average values of the telescope's normal and inverted positions yielded 120 elevation angles. During observations, each total station was carefully set up on a tripod to minimize movement. The maximum elevation angle was about 11°04', and the minimum was about -2°11'.

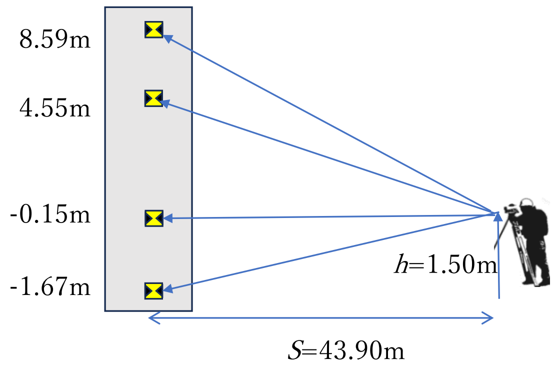

Figure 4. Elevation Angle Observations in Field Experiments

3.2 Angle Measurement Performance of the Total Station

3.2.1 Uncertainty in Horizontal Angle Sighting

As reported by Murakami (2024), the angle measurement uncertainty per sighting was 2.5 seconds, which was larger than the uncertainty derived from the inspection data.

3.2.2 Uncertainty in Elevation Angle Sighting

Since TS position was thought to change slightly when exchanging TS units, the mean and standard deviation of 10 elevation angles per TS per sighting point were calculated from the observation results to avoid the effect of change in TS position. Subsequently, the mean standard deviation across all TS units and all sighting points was computed. The result yielded a standard deviation of 3.6 seconds for the elevation angle observations, from which a standard deviation of 5.0 seconds per sighting was estimated. This is unexpectedly large compared to the standard deviation of 2.5 seconds per sighting for horizontal angle observations in the field.

4. Summary of Angle Measurement Performance and Challenges

During instrument inspections, the angular measurement uncertainty of TS instruments was investigated for both 1-second and 5-second readings.
The measurement uncertainty for horizontal angles was:
  1.1 seconds per sighting, in terms of standard deviation for 1-second reading;
  1.7 seconds per sighting, in terms of standard deviation for 5-second reading.
The measurement uncertainty for elevation angles was:
  1.8 seconds per sighting, in terms of standard deviation for 1-second reading;
  2.4 seconds per sighting, in terms of standard deviation for 5-second reading.

The measurement uncertainty for elevation angles is approximately 1.5 times that for horizontal angles.
In field experimental observations, the TS was set to 5-second reading for uncertainty investigation.
The measurement uncertainty for horizontal angles was:
 2.5 seconds per sighting, in terms of standard deviation.
The measurement uncertainty for elevation angles was:
 5.0 seconds per sighting, in terms of standard deviation.
The measurement uncertainty for elevation angles is twice that for horizontal angles.
The reasons why the measurement uncertainty for elevation angles is larger than that for horizontal angles, and why the measurement uncertainty in the experimental observations is larger for both elevation and horizontal angles than that in the instrument inspections, are currently under investigation.
Speculations regarding the cause include the following.

- During instrument inspections, observers *carefully* align their sights on the clear target of the collimator's reticle scale. In contrast, during field experiments, observers *quickly* align their sights on a target plate of a certain size, just as they would during practical work. For this reason, instrument inspections are expected to show a smaller standard deviation.
- The manufacturer's catalog for the TS does not distinguish between horizontal and vertical directions for the telescope's resolution and minimum reading values, making it unlikely that the instrument itself is the cause.
- Individual differences in the observer's sighting *did* appear in the elevation angle observations. The standard deviations of two out of three observers (the observer with the maximum standard deviation and the observer with the minimum standard deviation) showed a significant difference via F-test, suggesting the possibility of individual variation appearing in the vertical direction. However, even after removing the data from the observer with the maximum standard deviation, the standard deviation per sighting for elevation angles was 4.5 seconds, which is still larger than that for horizontal angles.
- In field experiments, the distance to the target for horizontal angle observations is 150-200m, while for elevation angle observations it is 44 m, representing a significant difference. However, the empirical rule that aiming is easier at closer distance does not explain why the standard deviation of elevation angle observations is larger. Furthermore, in instrument inspections, the collimator's reticle scale is set to infinity, so there is no effect in distance between horizontal and elevation angle observations.

The above speculations require confirmation based on data in a future study.

ACKNOWLEDGEMENTS

The author thanks the members of the study group on modernizing the operation specification for public surveys for their advice and discussion in this study. He also thanks the staff members of the Surveying Technology Center for their conducting experimental observations.

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BIOGRAPHICAL NOTES

Mr. MURAKAMI Masaki has been Vice President of the Japan Association of Surveyors since 2019. He also works as President of Japan Federation of Surveyors. He worked for the Geospatial Information Authority of Japan and retired in 2015 as Deputy Director General. He works for Pasco corporation after retirement.

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